XXIII.

ON THE REDUCTION OF DIFFERENT STAR CATALOGUES TO A COMMON SYSTEM.

By William A. Rogers.

Presented May 9th, 1883.

The communication of Professor Safford, in the March number of the Monthly Notices of the Royal Astronomical Society, cites in a forcible way some of the causes of discrepancies in stellar co-ordinates to which too little attention has hitherto been paid. Incidentally he refers to the class of errors which are introduced in the computation of the systematic corrections necessary to reduce different catalogues to a common system. Without this reduction it is impossible to obtain the element proper motion with the degree of precision which modern observations call for.

It is unfortunate that this necessity exists, since considerable uncertainty must always remain in the determination of these corrections. One of the serious demands of instrumental astronomy at the present time is the independent determination at a few widely separated observatories of all the elements which define stellar positions, without direct reference to any assumed fundamental system. The Catalogue of A. G. Publication XIV. by Dr. Auwers is probably more nearly free from systematic errors than any hitherto constructed; but the independent researches of Professor Boss show that the fundamental observations in declination from about 1815 to 1845 differ as a system from the Auwers-Bradley system by an amount which cannot be neglected.

Since it is obviously impracticable to redetermine the instrumental constants with which the different catalogues to be compared have been constructed, by a direct reference to the fundamental system to which they are to be referred, and with these constants to deduce new co-ordinates, we must seek the best method of deriving the systematic

corrections required. The general tendency of modern practice is towards the graphic method, in preference to a rigid analytical determination. But the difficulty exists that computers do not agree in the details of the graphical methods employed, and hence with the same data different results are obtained. It is the common practice to draw the curves which represent the residuals in right ascension with this function as the horizontal argument, but there are several catalogues in which the residuals $\Delta\alpha$ are functions of both the right ascension and of the declination.

In the choice of any method, it is obvious that preference must be given to that in which the residuals $\Delta\alpha$ and $\Delta\delta$ are reduced to a minimum, *whatever the order or the limits of the groups into which they may be divided.* It is believed that this will be best accomplished by the use of what, for the lack of a better term, may be described as double-argument curves.

Suppose, for example, that all the residuals in right ascension for a given catalogue which fall within the limits of the groups in declination $-10°\ldots+0°$, $+0°\ldots+10°$, $+10°\ldots+15°$, etc., are arranged in groups in the order of right ascension. Since the values of the residuals may be assumed to be true (nearly) for the mean of the groups into which they are divided, whether the argument be the right ascension or the declination, it is obvious that we can choose either the values which are functions of the right ascension, or those which depend on the declination, at pleasure, in the construction of the graphic curves. But whichever is chosen, the residuals derived from the constructed curves no longer represent the mean values which correspond to the other argument; that is, the values which are functions of one argument are derived from the same curve, while those which are functions of the other argument are derived from different curves.

The character of these double-argument functions are illustrated in the following (fictitious) example.

$\Delta\alpha$

TABLE I.

	−10° .. +0°	+0° .. +10°	+10° .. +20°	+20° .. +30°	+30° .. +40°	+40° .. +50°	+50° .. +60°	+60° .. +70°
h. h.	s.	s.	s.	s.	s.	s.	s.	s.
0.. 2	+.050	+.129	+.175	+.232	+.323	+.338	+.413	+.427
2.. 4	+.077	+.076	+.118	+.213	+.218	+.300	+.300	+.334
4.. 6	+.031	+.064	+.110	+.118	+.193	+.194	+.214	+.216
6.. 8	+.043	+.058	+.050	+.088	+.077	+.112	+.100	+.125
8..10	+.028	+.028	+.044	+.047	+.064	+.058	+.061	+.062
10..12	+.006	+.013	+.007	+.013	+.017	+.015	+.016	+.017
12..14	−.007	−.019	−.009	−.021	−.018	−.030	−.034	−.037
14..16	−.008	−.041	−.058	−.038	−.082	−.085	−.090	−.100
16..18	−.041	−.049	−.066	−.080	−.087	−.115	−.140	−.140
18..20	−.030	−.083	−.118	−.091	−.148	−.150	−.190	−.213
20..22	−.069	−.097	−.113	−.138	−.163	−.200	−.225	−.250
22..24	−.068	−.124	−.150	−.150	−.215	−.230	−.265	−.300

These residuals may be considered as representing the true deviation from the normal places, either at 1^h, 3^h, 5^h, etc., or at −5°, +5°, +15°, etc.

If the right ascension is assumed as the argument, we shall have from the curves represented by $\Delta\alpha$ the following: —

TABLE II.

	−5°	+5°	+15°	+25°	+35°	+45°	+55°	+65°
h.	s.	s.	s.	s.	s.	s.	s.	s.
1	+.072	+.106	+.163	+.235	+.290	+.350	+.415	+.425
3	+.062	+.087	+.130	+.180	+.232	+.280	+.300	+.322
5	+.050	+.072	+.097	+.130	+.170	+.200	+.207	+.220
7	+.037	+.053	+.060	+.087	+.112	+.130	+.130	+.128
9	+.027	+.033	+.033	+.047	+.056	+.067	+.067	+.060
11	+.013	+.014	+.014	+.016	+.010	+.018	+.018	+.011
13	−.009	−.006	−.007	−.018	−.030	−.032	−.037	−.040
15	−.020	−.026	−.025	−.047	−.060	−.079	−.087	−.090
17	−.033	−.045	−.045	−.073	−.093	−.117	−.137	−.143
19	−.048	−.070	−.071	−.105	−.123	−.157	−.186	−.205
21	−.060	−.100	−.120	−.133	−.150	−.197	−.226	−.257
23	−.077	−.127	−.127	−.160	−.165	−.233	−.260	−.306

The values in the horizontal columns, having been derived from different curves, no longer sustain the relation which existed in the corresponding values of Table I. They may be connected symmetrically by drawing a series of smooth curves with the declination for the argument, as shown in Table III.

TABLE III.

	−5°	+5°	+15°	+25°	+35°	+45°	+55°	+65°
h.	s.	s.	s.	s.	s.	s.	s.	s.
1	+.053	+.112	+.170	+.228	+.288	+.347	+.407	+.452
3	+.048	+.092	+.138	+.182	+.228	+.270	+.307	+.333
5	+.044	+.075	+.103	+.131	+.168	+.196	+.213	+.228
7	+.035	+.054	+.072	+.092	+.109	+.127	+.130	+.130
9	+.026	+.036	+.041	+.050	+.054	+.060	+.066	+.070
11	+.013	+.013	+.012	+.012	+.013	+.013	+.013	+.012
13	−.007	−.008	−.013	−.022	−.028	−.036	−.040	−.041
15	−.023	−.027	−.036	−.048	−.067	−.077	−.087	−.097
17	−.041	−.047	−.059	−.077	−.098	−.120	−.133	−.145
19	−.057	−.068	−.083	−.103	−.126	−.157	−.180	−.200
21	−.076	−.090	−.108	−.132	−.157	−.194	−.227	−.267
23	−.107	−.117	−.139	−.167	−.190	−.220	−.264	−.317

Nearly the same results should be reached by starting with the declination for the argument, using the values of $\Delta\alpha$ in the horizontal columns. With the corrected values thus obtained, curves representing these residuals were drawn with the right ascension as the argument.

Proceeding in this order, we obtain finally the values of $\Delta\alpha$ given in

TABLE IV.

	−5°	+5°	+15°	+25°	+35°	+45°	+55°	+65°
h.	s.	s.	s.	s.	s.	s.	s.	s.
1	+.050	+.117	+.185	+.245	+.310	+.355	+.388	+.418
3	+.043	+.092	+.145	+.194	+.238	+.268	+.296	+.316
5	+.036	+.073	+.109	+.138	+.168	+.188	+.208	+.224
7	+.030	+.058	+.073	+.086	+.098	+.110	+.126	+.127
9	+.025	+.033	+.038	+.040	+.043	+.047	+.050	+.052
11	+.013	+.012	+.009	+.006	+.000	−.004	−.006	−.007
13	−.004	−.009	−.016	−.023	−.037	−.042	−.047	−.054
15	−.019	−.028	−.040	−.053	−.067	−.080	−.098	−.120
17	−.036	−.048	−.064	−.080	−.099	−.118	−.136	−.153
19	−.054	−.070	−.088	−.107	−.134	−.160	−.183	−.200
21	−.066	−.090	−.114	−.137	−.167	−.188	−.213	−.240
23	−.086	−.114	−.142	−.166	−.198	−.228	−.264	−.300

It will be seen that there is a subtantial agreement between the values in Tables III. and IV. These values have been derived directly from the curves, and no attempt has been made to smooth them by the differences. Inasmuch as it will rarely, if ever, happen that the periodicity in both directions is as great as in this example, the agreement of the values may be taken as an index of the magnitude of the errors likely to be introduced through the process of drawing the

curves. It will not escape attention, that in this case, at least, the values in the two tables differ systematically at certain points. It is the experience of the writer that the periodicity thus introduced sometimes amounts to one fourth or one fifth as much as the systematic deviation of the catalogue compared from the normal system.

In the paper on "A Comparison of the Harvard College Observatory Catalogue of Stars for 1875.0 with the Fundamental Systems of Auwers, Safford, Boss, and Newcomb," the method here described has been essentially followed. But in order to reduce the magnitude of the residuals with which we have to deal, the computed corrections depending on both the right ascension and the declination were subtracted from the original residuals, and the values of $d\Delta\alpha$ and $d\Delta\delta$ which remain were treated in the way above described, giving the corrections found in Table III. Notwithstanding the criticism of Professor Safford that this method is unusual, I must maintain that it gives nearer approximations to the true corrections than can be obtained in any other way.

Professor Safford has given an exhaustive discussion of the most probable values of the right ascensions of a list of stars given in the Memoir to which reference has been made. This discussion involves, however, a knowledge of the systematic errors of the catalogues compared. It will be interesting to compare with his results the final results of the Harvard College observations of these stars, in which there is a direct dependence upon the system of Publication XIV., and in which, therefore, there is no need of applying corrections for a systematic deviation from the assumed system. An experience of thirteen years has shown that the Harvard College Meridian Circle can be relied upon to follow very closely the fundamental system chosen in a series of differential observations.

Two stars of the list have erroneous values in the final catalogue.

For β Lyræ, the right ascension should read $18^h\ 45^m\ 27^s.851$, instead of $27^s.774$.

In the case of υ Pegasi, there is a misprint in the volume for 1872 of $23^h\ 18^m\ 59^s.374$ for $59^s.574$.

By a reference to the original manuscripts it is found that both of these errors had been corrected, but by some mistake the corrections were not made on the sheets prepared for the printer. There were about a dozen errors of this kind, but all of them seem to have been corrected in printing except these two.

For the remaining stars the deviations from the positions given in Publication XIV. are given below, except for the star 1 H. Draconis,

where the comparison is made with the place derived from the correction given by Auwers in A. G. 1879, p. 2. These results are given in advance of publication, with the permission of Professor Pickering, the Director of the Observatory.

Year of Obs.	H. C. O. *minus* Pub. XIV.
Br. 6. $\alpha = 0^h\ 9^m$ $\delta = +76°.3$	s.
1871	—.04
"	+.00
"	+.11
"	+.18
1872	+.05
"	—.11
"	+.07
"	+.11
1873	+.11
"	+.19
"	+.12
"	+.11
"	+.33
"	+.14
"	+.30
"	—.05
1874	+.11
"	+.00
"	+.14
"	+.17
"	+.41
1875	+.49
"	+.31
1876	+.02
"	—.11
"	+.09
"	+.26
1877	+.12
"	+.05
"	+.19
"	+.51
"	+.47
"	+.42
"	+.29
"	+.14
η Cassiop. $\alpha = 0^h\ 42^m$ $\delta = +57°.2$	s.
1870	+.01
"	+.03
"	—.02
1871	+.06
"	—.01
"	+.02
"	+.18
"	+.07
"	+.05
1872	+.03
"	+.12
"	+.11
1873	+.18
"	+.13
"	+.13
"	+.14
"	+.12
1874	+.03
"	+.11
"	+.02
"	+.10
"	+.15
"	+.01
"	—.03
"	+.08
1876	+.05
"	+.07
"	+.08
"	+.08
1877	+.09
"	+.12
"	+.17
"	—.01
"	+.04
"	+.17
"	+.03
36 Camelop. $\alpha = 6^h\ 0^m$ $\delta = +65°.7$	s.
1871	+.17
"	+.08
"	—.03
1874	+.05
"	+.01
1876	s. +.07
"	+.10
"	+.15
"	+.00
1 H. Draconis. $\alpha = 9^h\ 19^m$ $\delta = +81°.9$	s.
1871 U.C.	+.25
"	+.17
1871 L.C.	+.07
"	+.14
"	+.18
"	+.01
"	—.06
"	+.12
"	+.19
"	+.19
"	+.15
"	+.00
"	+.04
"	+.03
1872 U.C.	+.14
"	—.09
"	+.26
"	—.02
"	+.19
"	+.52
"	+.23
"	—.11
"	+.17
1873 U.C.	+.25
1873 L.C.	+.15
"	+.28
"	—.01
"	+.33
"	+.34
"	+.35
1874 L.C.	+.25
"	+.34
"	+.42
"	+.49
1875 U.C.	+.38
"	+.28
1875 U.C.	s. +.19
"	+.22
"	+.31
"	+.52
"	[+.80]
"	+.18
1876 U.C.	+.55
"	+.06
"	+.01
1877 U.C.	+.69
"	—.26
"	+.35
"	+.11
"	+.28
1878 U.C.	+.12
"	+.17
"	+.63
"	+.03
"	+.21
Gr. 2164. $\alpha = 14^h\ 48^m$ $\delta = +59°.8$	s.
1871	+.01
"	+.18
"	—.06
"	+.08
"	+.04
"	+.02
"	+.05
"	+.06
"	—.06
1872	+.02
"	+.03
"	—.03
"	+.04
1873	+.05
"	—.03
"	—.06
"	—.01
"	—.06
"	—.11
"	—.10
"	—.12
"	—.28
1876	+.02

Year of Obs.	H. C. O. *minus* Pub. XIV.
θ Draconis	
$\alpha = 16^h\ 0^m$	
$\delta = +58°.9$	
	s.
1871	—.20
"	—.14
"	—.08
"	—.08
"	—.07
"	—.19
"	—.12
1872	—.07
"	—.06
"	—.09
"	—.10
"	—.04
"	—.11
1873	—.12
"	—.09
"	—.04
"	—.10
"	—.07
"	—.13
"	—.21
1874	+.08
"	—.11
"	—.01
"	+.00
"	+.08
"	—.01
"	+.00
"	+.00
"	+.02
"	—.18
"	+.03
1875	—.08
"	+.27
"	—.18
"	—.12
"	—.04
1876	—.10
"	—.35
"	—.04
"	—.28
"	—.14
"	—.07

Year of Obs.	H. C. O. *minus* Pub. XIV.
η Draconis.	
$\alpha = 16^h\ 22^m$	
$\delta = +61°.8$	
	s.
1871	+.03
"	—.21
"	—.17
1872	—.17
"	—.18
"	—.12
"	—.20
"	—.15
"	—.20
"	—.16
"	—.06
1873	+.12
"	—.13
"	—.14
"	—.15
"	—.08
"	—.39
1874	—.13
"	—.32
"	—.11
"	—.13
"	—.13
"	—.11
"	—.14
"	—.16
"	—.10
"	—.07
"	—.09
"	—.24
"	—.18
"	—.05
"	—.08
"	—.29
"	—.08
"	—.02
1875	—.08
"	—.18
"	—.16
"	—.19
"	—.19
"	—.24
"	—.25
"	—.02

Year of Obs.	H. C. O. *minus* Pub. XIV.
	s.
1875	—.16
"	—.13
"	—.15
1876	—.28
"	—.16
"	—.24
"	—.40
"	—.13
1877	—.24
"	—.28
"	—.05
"	—.30
"	—.36
1878	—.10
"	—.17
"	—.08
"	—.34
"	—.23
"	—.25
"	—.03
"	—.21
Gr. 2377.	
$\alpha = 16^h\ 43^m$	
$\delta = +57°.0$	
	s.
1871	+.09
"	—.11
"	—.10
"	—.08
"	—.12
"	—.13
1872	—.12
"	—.08
"	—.08
"	—.17
"	—.11
"	—.07
"	—.20
"	—.10
"	—.06
1873	—.16
"	—.25
"	—.06
"	—.03
"	—.08
"	—.20

Year of Obs.	H. C. O. *minus* Pub. XIV.
	s.
1874	+.04
"	+.02
"	+.05
"	—.11
"	—.09
"	—.10
"	—.07
"	—.15
"	+.04
"	—.13
1875	—.10
"	—.12
"	—.10
"	—.06
"	—.17
"	—.15
"	—.13
1876	+.03
"	—.15
"	—.13
"	—.18
"	—.17
"	—.12
Gr. 2900.	
$\alpha = 19^h\ 29^m$	
$\delta = +79°.4$	
	s.
1871	+.34
1872	+.20
"	+.07
"	+.07
1873 u.c.	+.11
"	+.02
"	+.16
"	—.04
"	+.04
"	+.17
"	—.02
"	+.24
"	+.15
"	+.37
"	+.09
1874	+.12
"	+.34
"	+.38
"	+.20
"	+.11

Collecting the results for each year we have: —

Br. 6.			1 H. Draconis.			η Draconis.		
Year.	$\Delta\alpha$	No. Obs.	Year.	$\Delta\alpha$	No. Obs.	Year.	$\Delta\alpha$	No. Obs.
	s.			s.			s.	
1871	+.062	4	1871	+.106	14	1871	—.117	3
1872	+.030	4	1872	+.143	9	1872	—.155	8
1873	+.156	8	1873	+.242	7	1873	—.128	6
1874	+.166	5	1874	+.375	4	1874	—.135	18
1875	+.400	2	1875	+.297	7	1875	—.159	11
1876	+.065	4	1876	+.207	3	1876	—.242	5
1877	+.274	8	1877	+.234	5	1877	—.246	5
			1878	+.232	5	1878	—.176	8
η Cassiop.			Gr. 2164.			Gr. 2377.		
	s.			s.			s.	
1870	+.007	3	1871	+.036	9	1871	—0.75	6
1871	+.062	6	1872	+.015	4	1872	—.110	9
1872	+.087	3	1873	—.080	9	1873	—.130	6
1873	+.140	5	1876	+.020	1	1874	—.050	10
1874	+.059	8				1875	—.083	7
1876	+.070	4	θ Draconis.			1876	—.119	6
1877	+.087	7		s.				
36 Camelop.			1871	—.126	7	Gr. 2900.		
	s.		1872	—.078	6		s.	
1871	+.073	3	1873	—.109	7	1871	+.340	1
1874	+.030	2	1874	—.009	11	1872	+.113	3
1876	+.080	4	1875	—.030	5	1873	+.117	11
			1876	—.163	6	1874	+.230	5

Finally we have the following values for H. C. O. *minus* Pub. XIV. The residuals taken from the Memoir quoted, are also given. It will be remembered that they were derived from the observations of 1871, 1872, 1874, and 1875, by applying the systematic corrections given in Tables I., II., and III.

Star.	$\Delta\alpha$ from Observations between 1870 and 1878.	$\Delta\alpha$ from Memoir.	Diff.
	s.	s.	s.
Br. 6	+.163	+.170	—.007
η Cassiop.	+.077	+.090	—.013
36 Camelop.	+.067	+.100	—.033
1 H. Draconis	+.204	+.192	+.012
Gr. 2164	—.014	+.087	—.101
θ Draconis	—.080	—.093	+.013
η Draconis	—.164	—.141	—.023
Gr. 2377	—.097	—.093	—.004
Gr. 2900	+.156	+.156	+.000

Aside from the two stars at first noted, it appears, therefore, that with the exception of Gr. 2164 the corrections given in my Memoir are substantiated by the entire series of Harvard College Observatory observations made between 1871 and 1878 inclusive.

HARVARD COLLEGE OBSERVATORY,
May 9, 1883.